Mid-Chapter Quiz: Lessons 4-1 through 4-4

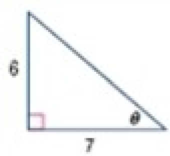
Find the exact values of the six trigonometric functions of θ.

1. 

SOLUTION:

The length of the side opposite θ is 24, the length of the side adjacent to θ is 10, and the length of the hypotenuse is 26.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{24}{26} \text{ or } \frac{12}{13}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{10}{26} \text{ or } \frac{5}{13}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{24}{10} \text{ or } \frac{12}{5}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{26}{24} \text{ or } \frac{13}{12}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{26}{10} = \frac{13}{5}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{10}{24} \text{ or } \frac{5}{12}$$

2. 

SOLUTION:

The length of the side opposite θ is 6, the length of the side adjacent to θ is 7, and the length of the hypotenuse is $\sqrt{85}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{6}{\sqrt{85}} \text{ or } \frac{6\sqrt{85}}{85}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{7}{\sqrt{85}} \text{ or } \frac{7\sqrt{85}}{85}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \text{ or } \frac{6}{7}$$
$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \text{ or } \frac{\sqrt{85}}{6}$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} \text{ or } \frac{\sqrt{85}}{7}$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} \text{ or } \frac{7}{6}$$

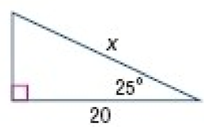
Find the value of *x*. Round to the nearest tenth if necessary.

3. 

SOLUTION:

An acute angle measure and the length of the side opposite the angle are given, so the tangent function can be used to find the length of the side adjacent to θ.

$$\begin{aligned} \tan\theta &= \frac{\text{opp}}{\text{adj}} \\ \tan 70° &= \frac{12}{x} \\ x\tan 70° &= 12 \\ x &= \frac{12}{\tan 70°} \\ x &\approx 4.4 \end{aligned}$$

4. 

SOLUTION:

An acute angle measure and the length of the side adjacent the angle are given, so the cosine function can be used to find the length of the hypotenuse.

$$\begin{aligned} \cos\theta &= \frac{\text{adj}}{\text{hyp}} \\ \cos 25° &= \frac{20}{x} \\ x\cos 25° &= 20 \\ x &= \frac{20}{\cos 25°} \\ x &\approx 22.1 \end{aligned}$$

Mid-Chapter Quiz: Lessons 4-1 through 4-4

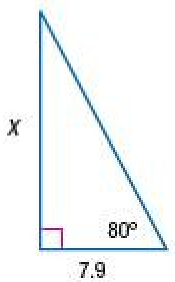
5. **SHADOWS** A pine tree casts a shadow that is 7.9 feet long when the Sun is 80° above the horizon.

a. Find the height of the tree.

b. Later that same day, a person 6 feet tall casts a shadow 6.7 feet long. At what angle is the Sun above the horizon?

SOLUTION:

a. Make a diagram of the situation.



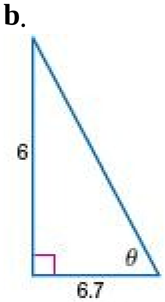
$$\tan 80 = \frac{x}{7.9}$$

$$7.9 \tan 80 = x$$

$$44.8 \approx x$$

The tree is about 45 feet high.

b.

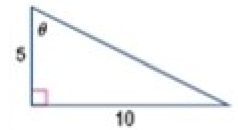


$$\tan \theta = \frac{6}{6.7}$$

$$\theta = \tan^{-1}\left(\frac{6}{6.7}\right)$$

$$\theta \approx 42°$$

Find the measure of angle θ. Round to the nearest degree if necessary.

6. 

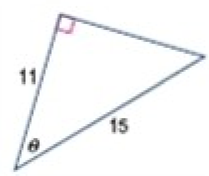
SOLUTION:

Because the lengths of the sides opposite and adjacent to θ are given, the tangent function can be used to find θ.

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{10}{5}$$

$$\theta = \tan^{-1} 2 \text{ or about } 63°$$

7. 

SOLUTION:

Because the length of the side adjacent to θ and the hypotenuse are given, the cosine function can be used to find θ.

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos \theta = \frac{11}{15}$$

$$\theta = \cos^{-1} \frac{11}{15} \text{ or about } 43°$$

8. Write $\frac{2\pi}{9}$ in degrees.

SOLUTION:

To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

$$\frac{2\pi}{9} = \frac{2\pi}{9} \text{ radians}$$

$$= \frac{2\pi}{9} \text{ radians}\left(\frac{180°}{\pi \text{ radians}}\right)$$

$$= 40°$$

Mid-Chapter Quiz: Lessons 4-1 through 4-4

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

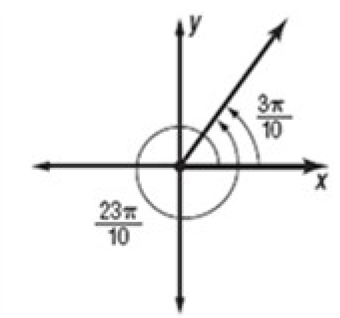
9. $\frac{3\pi}{10}$

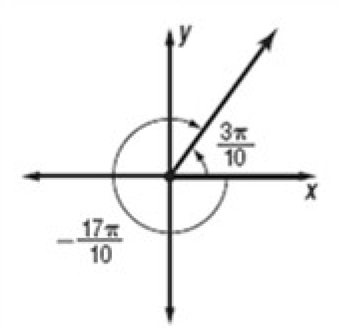
SOLUTION:

All angles measuring $\frac{3\pi}{10} + 2n\pi$ are coterminal with a $\frac{3\pi}{10}$ radian angle.

Sample answer: Let n = 1 and −1.

$$\frac{3\pi}{10} + 2(1)\pi = \frac{3\pi}{10} + 2\pi \text{ or } \frac{23\pi}{10}$$
$$\frac{3\pi}{10} - 2(1)\pi = \frac{3\pi}{10} - 2\pi \text{ or } -\frac{17\pi}{10}$$





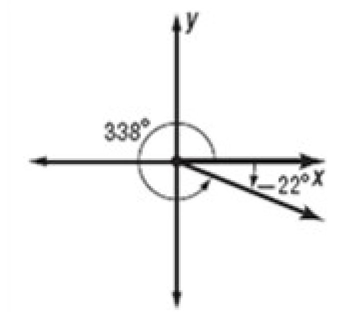
10. −22°

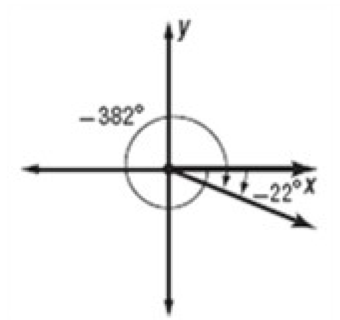
SOLUTION:

All angles measuring $-22° + 360n°$ are coterminal with a −22° angle.

Sample answer: Let n = 1 and −1.

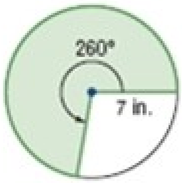
$$-22° + 360(1)° = -22° + 360° \text{ or } 338°$$
$$-22° + 360(-1)° = -22° - 360° \text{ or } -382°$$





Mid-Chapter Quiz: Lessons 4-1 through 4-4

11. **MULTIPLE CHOICE** Find the approximate area of the shaded region.



A 12.2 in^2
B 42.8 in^2
C 85.5 in^2
D 111.2 in^2

SOLUTION:

$$\text{Area} = \frac{260}{360}\pi(7)^2$$
$$= \frac{13}{18}(49\pi)$$
$$\approx 111.2$$

The correct choice is D.

12. **TRAVEL** A car is traveling at a speed of 55 miles per hour on tires that measure 2.6 feet in diameter. Find the approximate angular speed of the tires in radians per minute.

SOLUTION:

The linear speed is 55 miles per hour. Convert this to feet per minute.

$$\frac{55\text{ miles}}{\text{hour}} \cdot \frac{5280\text{ feet}}{1\text{ mile}} \cdot \frac{1\text{ hour}}{60\text{ minutes}}$$
$$= \frac{4840\text{ feet}}{\text{minute}}$$

The tires are 2.6 feet in diameter, so one rotation will be a distance of 2.6π feet. Convert the speed to rotations per minute.

$$\frac{4840\text{ feet}}{\text{minute}} \cdot \frac{1\text{ rotation}}{2.6\pi\text{ feet}} = \frac{4840}{2.6\pi}\text{rotations per minute}$$

Each rotation measures 2π radians. Convert rotations to radians.

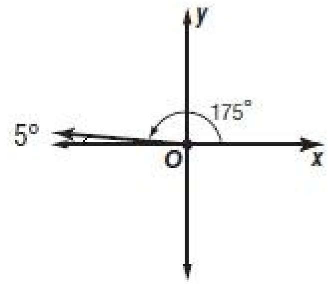
$$\frac{4840\text{ rotations}}{2.6\pi\text{ minutes}} \cdot \frac{2\pi\text{ radians}}{1\text{ rotation}} \approx 3723\text{ radians per minute}$$

Sketch each angle. Then find its reference angle.

13. $175°$

SOLUTION:

The terminal side of $175°$ lies in Quadrant II. Therefore, its reference angle is $\theta' = 180° - 175°$ or $5°$.

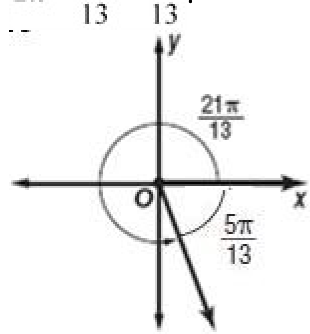


14. $\frac{21\pi}{13}$

SOLUTION:

The terminal side of $\frac{21\pi}{13}$ lies in Quadrant IV.

Therefore, its reference angle is $\theta' =$ $2\pi - \frac{21\pi}{13} = \frac{5\pi}{13}$.



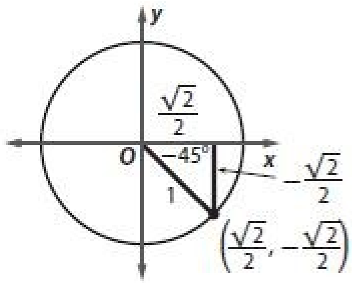
Mid-Chapter Quiz: Lessons 4-1 through 4-4

Find the exact value of each expression. If undefined, write *undefined*.

15. cos 315°

SOLUTION:

Because the terminal side of θ lies in Quadrant IV, the reference angle θ' is –45°.

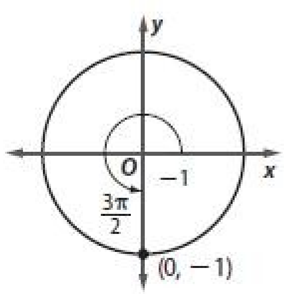


$$\cos(-45°) = \frac{\sqrt{2}}{2}$$

16. sec $\frac{3\pi}{2}$

SOLUTION:

Because the terminal side of θ lies on the negative *y*-axis, the reference angle θ' is 270°.



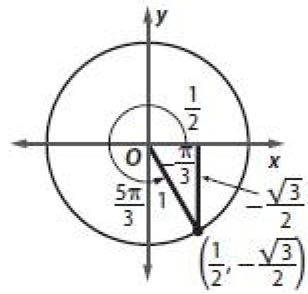
$$\sec 270° = \frac{1}{\cos 270°}$$

$$\sec 270° = \frac{1}{0} \text{ or undefined.}$$

17. sin $\frac{5\pi}{3}$

SOLUTION:

Because the terminal side of θ lies in Quadrant IV, the reference angle θ' is $-\frac{\pi}{3}$.

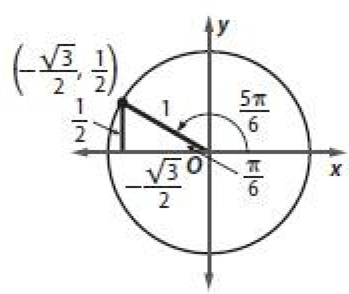


$$\sin\left(-\frac{\pi}{3}\right)^{\circ} = -\frac{\sqrt{3}}{2}$$

18. tan $\frac{5\pi}{6}$

SOLUTION:

Because the terminal side of θ lies in Quadrant II, the reference angle θ' is $\frac{\pi}{6}$.



$$\tan\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{3}$$

Mid-Chapter Quiz: Lessons 4-1 through 4-4

Find the exact values of the five remaining trigonometric functions of θ.

19. $\cos\theta = -\frac{2}{5}$, where $\sin\theta < 0$ and $\tan\theta > 0$

SOLUTION:

To find the other function values, you must find the coordinates of a point on the terminal side of θ. You know that $\cos\theta$ and $\sin\theta$ are negative, so θ must lie in Quadrant III. So, both x and y are negative.

Because $\cos\theta = \frac{x}{r}$ or $-\frac{2}{5}$, $x = -2$ and $r = 5$. Use the Pythagorean Theorem to find y.

$$\begin{aligned} y &= \pm\sqrt{r^2 - x^2} \\ &= \pm\sqrt{5^2 - (-2)^2} \\ &= \pm\sqrt{21} \end{aligned}$$

So, $y = -\sqrt{21}$.

Use $x = -2$, $y = -\sqrt{21}$, and $r = 5$ to write the five remaining trigonometric ratios.

$$\begin{aligned} \sin\theta &= \frac{y}{r} = -\frac{\sqrt{21}}{5} \\ \tan\theta &= \frac{y}{x} = \frac{\sqrt{21}}{2} \\ \csc\theta &= \frac{r}{y} = -\frac{5}{\sqrt{21}} \text{ or } -\frac{5\sqrt{21}}{21} \\ \sec\theta &= \frac{r}{x} = -\frac{5}{2} \\ \cot\theta &= \frac{x}{y} = \frac{2}{\sqrt{21}} \text{ or } \frac{2\sqrt{21}}{21} \end{aligned}$$

20. $\cot\theta = \frac{4}{3}$, where $\cos\theta > 0$ and $\sin\theta > 0$

SOLUTION:

To find the other function values, you must find the coordinates of a point on the terminal side of θ. You know that $\cos\theta$ and $\sin\theta$ are both positive, so θ must lie in Quadrant I. So, both x and y are positive.

Because $\cot\theta = \frac{x}{y} = \frac{4}{3}$, $x = 4$ and $y = 3$. Use the Pythagorean Theorem to find r.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{25} = 5 \end{aligned}$$

So, $r = 5$.

Use $x = 4$, $y = 3$, and $r = 5$ to write the five remaining trigonometric ratios.

$$\begin{aligned} \sin\theta &= \frac{y}{r} = \frac{3}{5} \\ \cos\theta &= \frac{x}{r} = \frac{4}{5} \\ \tan\theta &= \frac{y}{x} = \frac{3}{4} \\ \csc\theta &= \frac{r}{y} = \frac{5}{3} \\ \sec\theta &= \frac{r}{x} = \frac{5}{4} \end{aligned}$$

Mid-Chapter Quiz: Lessons 4-1 through 4-4

State the amplitude, period, frequency, phase shift, and vertical shift of each function. Then graph two full periods of the function.

21. $y = -3\sin\left(x - \frac{3\pi}{2}\right)$

SOLUTION:

In this function, $a = -3$, $b = 1$, $c = -\frac{3\pi}{2}$, and $d = 0$.

Because $d = 0$, there is no vertical shift.

Amplitude: $|a| = |-3|$ or 3

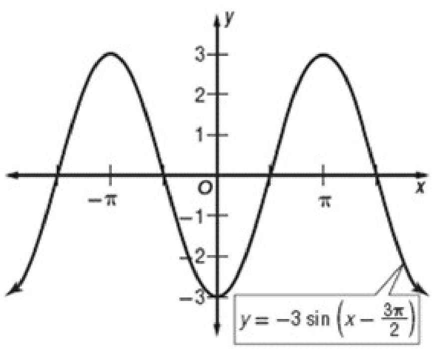
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|1|}$ or 2π

Frequency: $\frac{|b|}{2\pi} = \frac{|1|}{2\pi}$ or $\frac{1}{2\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{-\frac{3\pi}{2}}{|1|}$ or $\frac{3\pi}{2}$

Midline: $y = d$ or $y = 0$

Graph $y = -3 \sin x$ shifted $\frac{3\pi}{2}$ units to the right.



22. $y = 5 \cos 2x - 2$

SOLUTION:

In this function, $a = 5$, $b = 2$, $c = 0$, and $d = -2$.

Amplitude: $|a| = |5|$ or 6

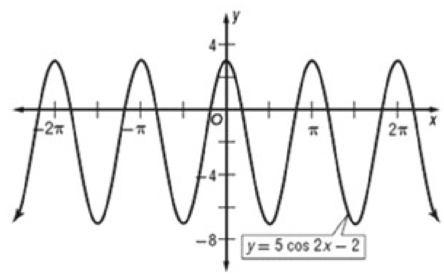
Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|2|}$ or π

Frequency: $\frac{|b|}{2\pi} = \frac{|2|}{2\pi}$ or $\frac{1}{\pi}$

Phase shift: $-\frac{c}{|b|} = -\frac{-0}{|1|}$ or 0

Midline: $y = d$ or $y = -2$

Graph $y = 5 \cos 2x$ shifted 2 units down.



23. **MULTIPLE CHOICE** Which of the functions has the same graph as $y = 3 \sin (x - \pi)$?

F $y = 3 \sin (x + \pi)$

G $y = 3\cos\left(x - \frac{\pi}{2}\right)$

H $y = -3 \sin (x - \pi)$

J $y = -3\cos\left(x + \frac{\pi}{2}\right)$

SOLUTION:

One method of solving would be to graph each function. Another method would be to compare points.

When $x = \frac{\pi}{2}$,

$$y = 3\sin(x-\pi)$$
$$y = 3\sin\left(\frac{\pi}{2}-\pi\right)$$
$$y = 3\sin\left(-\frac{\pi}{2}\right)$$
$$y = 3(-1)$$
$$y = -3$$

$$y = 3\cos\left(x-\frac{\pi}{2}\right)$$
$$y = 3\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\right)$$
$$y = 3\cos 0$$
$$y = 3$$
$$y = -3\sin(x-\pi)$$
$$y = -3\sin\left(\frac{\pi}{2}-\pi\right)$$
$$y = -3\sin\left(-\frac{\pi}{2}\right)$$
$$y = -3(-1)$$
$$y = 3$$
$$y = -3\cos\left(x+\frac{\pi}{2}\right)$$
$$y = -3\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)$$
$$y = -3\cos\pi$$
$$y = 3$$

Therefore, choices G, H, and J are incorrect.

While substituting one value into the equation for F will not guarantee that the equations are equivalent, the other choices have already been eliminated, so F is correct. Again, a graph of the equations will confirm this.

24. **SPRING** The motion of an object attached to a spring oscillating across its original position of rest can be modeled by $x(t) = A\cos\omega t$, where A is the initial displacement of the object from its resting position, ω is a constant dependent on the spring and the mass of the object attached to the spring, and t is time measured in seconds.

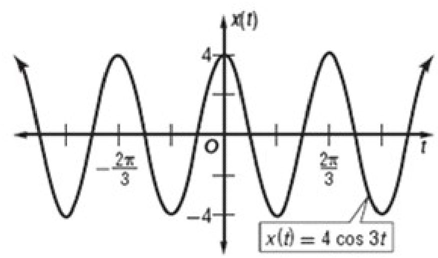
a. Draw a graph for the motion of an object attached to a spring and displaced 4 centimeters where ω = 3.

b. How long will it take for the object to return to its initial position for the first time?

c. The constant ω is equal to $\sqrt{\frac{k}{m}}$, where k is the *spring constant*, and m is the mass of the object. How would increasing the mass of an object affect the period of its oscillations? Explain your reasoning.

SOLUTION:

a. The displacement is 4, so the amplitude of the graph is also 4. The constant is 3, so $b = 3$ and the frequency is $\frac{3}{2\pi}$ in the graph. The period will be $\frac{2\pi}{3}$.



b. The period is $\frac{2\pi}{3}$. This is the amount of time it will take to go from one peak in the graph to the next peak. This is equivalent to about 2.1 seconds.

c. Sample answer: It will increase the period of the oscillations because as the mass m increases, the fraction $\frac{k}{m}$ decreases. The period of the mass' oscillations is $2\pi \div \sqrt{\frac{k}{m}}$, which increases as m increases.

When given a question like this, see what the increase causes. First, we increase m. This value is in the denominator of the fraction, so $\frac{k}{m}$ will decrease as m increases and k remains constant. The square root of a decreasing value also decreases, so $\sqrt{\frac{k}{m}}$ decreases as m increases.

The period is equal to $2\pi \div \sqrt{\frac{k}{m}}$. With everything else constant, when a value in the denominator is decreasing, the value of the expression will increase.

Therefore, the period of the mass' oscillations increases as *m* increases.

25. **BUOY** The height above sea level in feet of a signal buoy's transmitter is modeled by $h = a \sin bt + \frac{11}{2}$. In rough waters, the height cycles between 1 and 10 feet, with 4 seconds between cycles. Find the values of *a* and *b*.

SOLUTION:

The value of *a* is the amplitude, which is one half of the difference between the greatest height and the smallest height.

$$\frac{10-1}{2} = 4.5$$

The period is $\frac{2\pi}{b}$, which is 4 seconds.

$$\frac{2\pi}{b} = 4$$

$$2\pi = 4b$$

$$\frac{2\pi}{4} = b$$

$$\frac{\pi}{2} = b$$