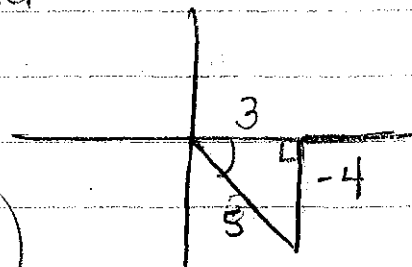


HW #47 p. 352. 1-35 odd

1. $\cos \theta = \frac{3}{5}$, $(270^\circ, 360^\circ)$
(QIV)



$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \left(\frac{-4}{5} \right) \left(\frac{3}{5} \right) = \frac{-24}{25}$$

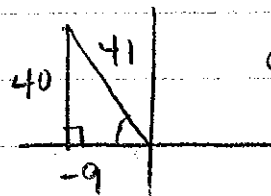
$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$= 2 \left(\frac{3}{5} \right)^2 - 1 = \frac{18}{25} - \frac{25}{25} = \frac{-7}{25}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(\frac{-4}{3} \right)}{1 - \left(\frac{-4}{3} \right)^2} = \frac{-8/3}{1 - 16/9}$$

$$= \frac{-8/3}{9/9 - 16/9} = \frac{-8/3}{-7/9} = \frac{-8}{3} \cdot \frac{-9}{7} = \frac{24}{7}$$

3. $\cos \theta = \frac{-9}{41}$ $(90^\circ, 180^\circ)$
QII



$$a^2 + 81 = 1681$$

$$a = 40$$

$$\sin 2\theta = 2 \left(\frac{40}{41} \right) \left(\frac{-9}{41} \right)$$

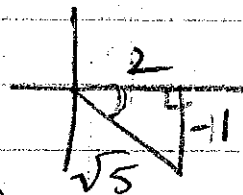
$$= \frac{-720}{1681}$$

$$\cos 2\theta = 2 \left(\frac{-9}{41} \right)^2 - 1 = 2 \left(\frac{81}{1681} \right) - 1 = \frac{162}{1681} - \frac{1681}{1681}$$

$$= \frac{-1519}{1681}$$

$$\tan 2\theta = \frac{2 \left(\frac{40}{-9} \right)}{1 - \left(\frac{40}{-9} \right)^2} = \frac{80}{-9} = \frac{80}{-9} = \frac{80 \cdot 81}{-9 \cdot 1519} = \frac{720}{1519}$$

$$5. \tan \theta = -\frac{1}{2} \quad \left(\frac{3\pi}{2}, 2\pi \right)$$

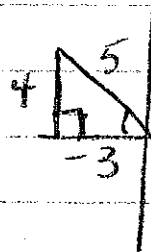


$$\sin 2\theta = 2 \left(\frac{-1}{\sqrt{5}} \right) \left(\frac{2}{\sqrt{5}} \right) = \frac{-4}{5}$$

$$\cos 2\theta = 2 \left(\frac{2}{\sqrt{5}} \right)^2 - 1 = 2 \left(\frac{4}{5} \right) - 1 = \frac{8}{5} - \frac{5}{5} = \frac{3}{5}$$

$$\tan 2\theta = \frac{2 \left(-\frac{1}{2} \right)}{1 - \left(-\frac{1}{2} \right)^2} = \frac{-1}{1 - \frac{1}{4}} = \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$$

$$7. \sin \theta = \frac{4}{5} \quad \left(\frac{\pi}{2}, \pi \right)$$



$$\sin 2\theta = 2 \left(\frac{4}{5} \right) \left(\frac{-3}{5} \right) = \frac{-24}{25}$$

$$\cos 2\theta = 2 \left(\frac{-3}{5} \right)^2 - 1 = 2 \left(\frac{9}{25} \right) - 1 = \frac{18}{25} - \frac{25}{25} = \frac{-7}{25}$$

$$\tan 2\theta = \frac{2 \left(\frac{4}{5} \right) \left(\frac{-3}{5} \right)}{1 - \left(\frac{4}{5} \right)^2} = \frac{-\frac{24}{25}}{1 - \frac{16}{25}} = \frac{-\frac{24}{25}}{\frac{9}{25}} = \frac{-24}{9} = \frac{-8}{3}$$

$$= \frac{-8}{3} \cdot \frac{-9}{7} = \frac{24}{7}$$

#47

9. $\sin 2\theta = \cos \theta$

$$2\sin \theta \cos \theta = \cos \theta$$

$$2\sin \theta \cos \theta - \cos \theta = 0$$

$$\cos \theta (2\sin \theta - 1) = 0$$

$$\cos \theta = 0 \quad 2\sin \theta - 1 = 0$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

11. $\cos 2\theta - \sin \theta = 0$

$$(1 - 2\sin^2 \theta) - \sin \theta = 0$$

$$-2\sin^2 \theta - \sin \theta + 1 = 0$$

$$2\sin^2 \theta + \sin \theta - 1 = 0$$

$$2x^2 + x - 1 = 0$$

$$x = \sin \theta$$

$$(2x - 1)(x + 1) = 0$$

$$2x - 1 = 0 \quad x + 1 = 0$$

$$x = \frac{1}{2} \quad x = -1$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -1$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad \theta = \frac{3\pi}{2}$$

$$13. \sin 2\theta \csc \theta = 1$$

$$(2 \sin \theta \cos \theta) \frac{1}{\sin \theta} = 1$$

$$2 \cos \theta = 1$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$15. a. 242 = \frac{88^2 \sin 2\theta}{32}$$

$$\frac{7744}{7744} = \frac{7744 \sin 2\theta}{7744}$$

$$\sin 90^\circ = 1$$

$$1 = \sin 2\theta$$

$$\frac{90^\circ}{2} = \frac{2\theta}{2}$$

$$\theta = 45^\circ$$

$$b. d = \frac{v_0^2 \cdot 2 \sin \theta \cos \theta}{32}$$

$$d = \frac{v_0^2 \sin \theta \cos \theta}{16}$$

$$17. \tan^3 \theta = \tan \theta \cdot \tan^2 \theta$$

$$= \tan \theta \cdot \left(\frac{1 - \cos 2\theta}{1 + \cos 2\theta} \right)$$

$$= \frac{\tan \theta - \tan \theta \cos 2\theta}{1 + \cos 2\theta}$$

$$\begin{aligned}
 19. \cot^3 \theta &= \cot \theta \cdot \cot^2 \theta \\
 &= \cot \theta \cdot \frac{\cos^2 \theta}{\sin^2 \theta} \\
 &= \cot \theta \cdot \left(\frac{1 + \cos 2\theta}{2} \right) = \frac{\cot \theta (1 + \cos 2\theta)}{1 - \cos 2\theta} \\
 &= \frac{\left(\frac{1 - \cos 2\theta}{2} \right)}{\left(\frac{1 - \cos 2\theta}{2} \right)} = \boxed{\frac{\cot \theta + \cot \theta \cos 2\theta}{1 - \cos 2\theta}}
 \end{aligned}$$

$$\begin{aligned}
 21. \sin^2 \theta \cos^3 \theta &= \left(\frac{1 - \cos 2\theta}{2} \right) \left(\frac{1 + \cos 2\theta}{2} \right) \cos \theta \\
 &= \frac{(1 - \cos^2 2\theta) \cos \theta}{4} \\
 &= \frac{1 - \left(\frac{1 + \cos 2(2\theta)}{2} \right)}{4} \cos \theta = \frac{\frac{2}{2} - \frac{1 + \cos 4\theta}{2}}{4} \cdot \cos \theta \\
 &= \frac{(1 - \cos 4\theta) \cos \theta}{4} = \frac{\cos \theta - \cos \theta \cos 4\theta}{4} \\
 &= \boxed{\frac{\cos \theta - \cos \theta \cos 4\theta}{8}}
 \end{aligned}$$

$$23. \frac{\sin^4 \theta}{\cos^2 \theta} = \frac{\sin^2 \theta \cdot \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\left(\frac{1-\cos 2\theta}{2}\right) \left(\frac{1-\cos 2\theta}{2}\right)}{\frac{1+\cos 2\theta}{2}}$$

$$= \frac{(1-\cos 2\theta)(1-\cos 2\theta)}{4} \left(\frac{2}{1+\cos 2\theta}\right)$$

$$\frac{1-2\cos 2\theta + \cos^2 2\theta}{2+2\cos 2\theta} = \frac{1-2\cos 2\theta + \left(\frac{1+\cos 4\theta}{2}\right)}{2+2\cos 2\theta}$$

$$= \frac{2-4\cos 2\theta + 1 + \cos 4\theta}{2} \cdot \frac{1}{2+2\cos 2\theta}$$

$$= \frac{3-4\cos 2\theta + \cos 4\theta}{4+4\cos 2\theta}$$

$$25. \cos^2 \theta - \frac{3}{2} \cos 2\theta = 0$$

$$\frac{1+\cos 2\theta}{2} - \frac{3\cos 2\theta}{2} = 0$$

$$\frac{1-2\cos 2\theta}{2} = 0$$

$$1-2\cos 2\theta = 0$$

$$-2\cos 2\theta = -1$$

$$\cos 2\theta = \frac{1}{2}$$

$$2\theta = \frac{\pi}{3} \quad 2\theta = \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$27. \cos^2 \theta - \sin \theta = 1$$

$$1 - \sin^2 \theta - \sin \theta = 1$$

$$\sin^2 \theta + \sin \theta = 0$$

$$\sin \theta (\sin \theta + 1) = 0$$

$$\sin \theta = 0 \quad \sin \theta + 1 = 0$$

$$\sin \theta = 0 \quad \Rightarrow \theta = 0, \pi$$

$$\sin \theta = -1 \quad \Rightarrow \theta = \frac{3\pi}{2}$$

$$29. \sin 67.5^\circ = \sin \frac{135^\circ}{2} \quad 67.5^\circ \text{ QI sine pos}$$

$$= \sqrt{\frac{1 - \cos 135^\circ}{2}} = \sqrt{\frac{1 - \frac{-\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2}{2} - \frac{-\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{2}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2 - \sqrt{2}}{4}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$$

$$31. \tan 157.5^\circ = \tan \frac{315^\circ}{2} \quad 157.5^\circ \text{ QII tan is neg.}$$

$$= \frac{1 - \cos 315^\circ}{\sin 315^\circ} = \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}$$

$$= \frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{-\sqrt{2}} = \frac{2 - \sqrt{2}}{\sqrt{2}} \cdot \frac{-\sqrt{2}}{-\sqrt{2}} = \frac{-2 + \sqrt{2}}{\sqrt{2}}$$

OR $\frac{-2\sqrt{2} + 2}{2}$
OR $1 - \sqrt{2}$

$$33. \quad \frac{\sin \theta}{2} + \frac{\cos \theta}{-2} = 1$$

$$\left(\frac{\pm \sqrt{1 - \cos \theta}}{2} \right)^2 = (1 - \cos \theta)^2$$

$$= \frac{1 - \cos \theta}{2} = 1 - 2\cos \theta + \cos^2 \theta$$

$$= \frac{1 - \cos \theta}{-1 + 2\cos \theta} = \frac{2 - 4\cos \theta + 2\cos^2 \theta}{-1 + \cos \theta}$$

$$x = \cos \theta$$

$$0 = 1 - 3\cos \theta + 2\cos^2 \theta$$

$$2x^2 - 3x + 1 = 0$$

$$(2x - 1)(x - 1) = 0$$

$$2x - 1 = 0 \quad x - 1 = 0$$

$$x = 1/2 \quad x = 1$$

$\cos \theta = 1/2$ $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$	$\cos \theta = 1$ $\theta = 0$
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$$35. \quad 2\sin \frac{\theta}{2} = \sin \theta$$

$$\left(2 \left(\frac{\pm \sqrt{1 - \cos \theta}}{2} \right) \right)^2 = (\sin \theta)^2$$

$$4 \left(\frac{1 - \cos \theta}{2} \right) = \sin^2 \theta$$

$$2(1 - \cos \theta) = 1 - \cos^2 \theta$$

$$2 - 2\cos \theta = 1 - \cos^2 \theta$$

$$\cos^2 \theta - 2\cos \theta + 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1 \quad x = 1$$

$$\cos \theta = 1$$

$\theta = 0$
