HW #4 p.31 #42, 43, 45, 47, 49, 59, 61-66

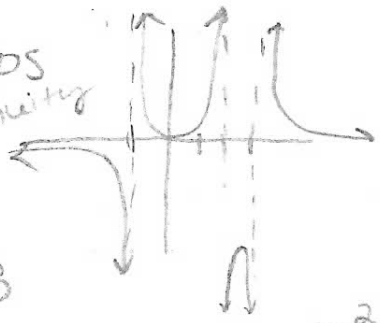
42. $x \neq 1$ infinite discontinuity (vertical asymptote)

$\lim_{x \to -\infty} g(x) = -2$ $\lim_{x \to \infty} g(x) = -2$

43. $x \neq 0$ infinite discontinuity (vertical asymptote)

$\lim_{x \to -\infty} h(x) = 0$ $\lim_{x \to \infty} h(x) = 0$

45. $f(x) = \dfrac{x^2}{x^3 - 4x^2 + x + 6}$ ← zeros ← discontinuity



infinite discontinuity at $x = -1$, $x = 2$, $x = 3$

$\lim_{x \to -\infty} f(x) = 0$ $\lim_{x \to \infty} f(x) = 0$

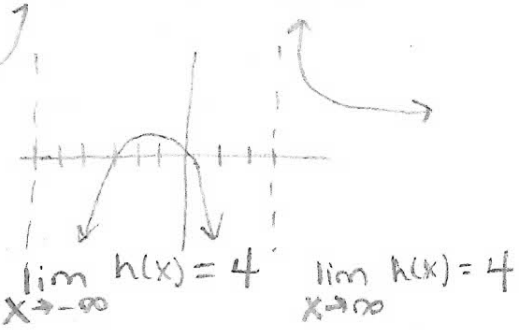
$x^2 = 0$

zero: $x = 0$

47. $h(x) = \dfrac{4x^2 + 11x - 3}{x^2 + 3x - 18}$ ← zeros

$(x+6)(x-3)$

infinite discontinuity at $x \neq -6$ $x = 3$



zeros: $x = -3, \frac{1}{4}$

$4x^2 + 11x - 3 = 0$

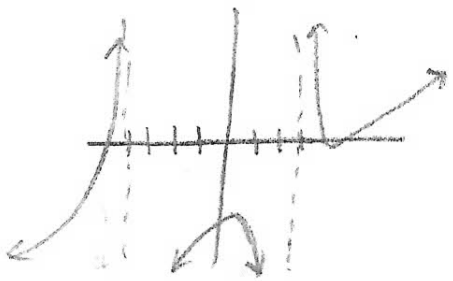
$(4x - 1)(x + 3) = 0$

$4x - 1 = 0$ $\quad x + 3 = 0$

$x = \frac{1}{4}$ $\quad x = -3$

Hint: To find pts of discontinuity, set den. = 0.

To find zeros set the numerator = 0.



49. $h(x) = \dfrac{x^3 - 5x^2 - 26x + 120}{x^2 + x - 12}$ ← zeros ← discontinuity

$(x+4)(x-3) = 0$

infinite discontinuity at $x = -4, 3$

$\lim_{x \to -\infty} h(x) = -\infty$ $\lim_{x \to \infty} h(x) = \infty$

find the intersection.

zeros: $x = -5, 4, 6$ $\underbrace{x^3 - 5x^2 - 26x + 120}_{Y_1} = \underbrace{0}_{Y_2}$

59. $f(x) = \dfrac{x^4}{x^5}$ → simplifies $f(x) = \dfrac{1}{x}$

infinite because there is a vertical asymptote at $x = 0$

61. $f(x) = \begin{cases} x^2 + a & \text{if } x \geq 3 \\ bx + a & \text{if } -3 < x < 3 \\ \sqrt{-b - x} & \text{if } x \leq -3 \end{cases}$

set equal when $x = 3$

set equal when $x = -3$

$3^2 + a = 3b + a$

$-a \quad\quad -a$

$\dfrac{9}{3} = \dfrac{3b}{3}$

$\boxed{3 = b}$

$3(-3) + a = \sqrt{-3 - (-3)}$

$-9 + a = \sqrt{0}$

$+9 \quad\quad +9$

$\boxed{a = 9}$

62. $\lim_{x \to -\infty} f(x) = -\infty$ b/c $f(-x) = f(x)$

63. $\lim_{x \to -\infty} f(x) = \infty$ b/c $f(-x) = -f(x)$

64. $\lim_{x \to -\infty} f(x) = -\infty$ b/c $f(-x) = -f(x)$

#4

65. $\lim_{x \to -\infty} f(x) = \infty$ b/c $f(-x) = f(x)$

66. Sample answer: (answers may vary)

$$f(x) = \frac{(x+2)(x+3)}{(x+2)}$$ removable discontinuity at $x = -2$

The discontinuity can be eliminated by redifining the function at the hole using a piecewise function

$$g(x) = \begin{cases} \frac{(x+2)(x+3)}{(x+2)} & \text{if } x \neq -2 \\ x + 3 & \text{if } x = -2 \end{cases}$$

The piecewise is defined for all x