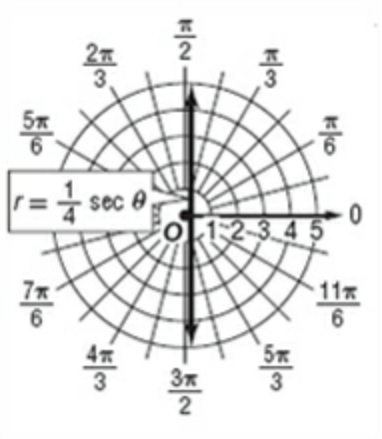
Mid-Chapter Quiz: Lessons 9-1 through 9-3

Graph each equation.

10. $r = \frac{1}{4} \sec \theta$



SOLUTION:

Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

θ	$r = \frac{1}{4}\sec\theta$
0	0.3
$\frac{\pi}{6}$	0.3
$\frac{\pi}{3}$	0.5
$\frac{\pi}{2}$	–
$\frac{2\pi}{3}$	−0.5
$\frac{5\pi}{6}$	−0.3
π	−0.3
$\frac{7\pi}{6}$	−0.3
$\frac{4\pi}{3}$	−0.5
$\frac{3\pi}{2}$	–
$\frac{5\pi}{3}$	0.5
$\frac{11\pi}{6}$	0.3
2π	0.3

Graph the ordered pairs (r, θ) and connect them with a line.

Mid-Chapter Quiz: Lessons 9-1 through 9-3

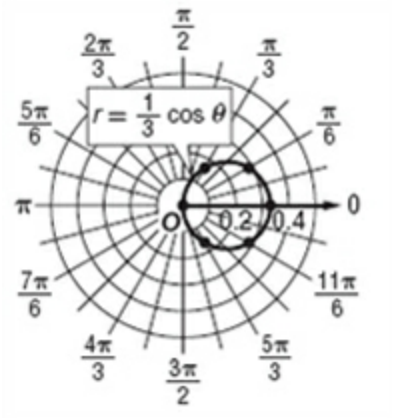
11. $r = \frac{1}{3}\cos\theta$

SOLUTION:

Because the polar equation is a function of the cosine function, it is symmetric with respect to the polar axis. Therefore, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = \frac{1}{3}\cos\theta$
0	0.3
$\frac{\pi}{6}$	0.3
$\frac{\pi}{4}$	0.2
$\frac{\pi}{3}$	0.2
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	−0.2
$\frac{3\pi}{4}$	−0.2
$\frac{5\pi}{6}$	−0.3
π	−0.3

Use these points and polar axis symmetry to graph the function.



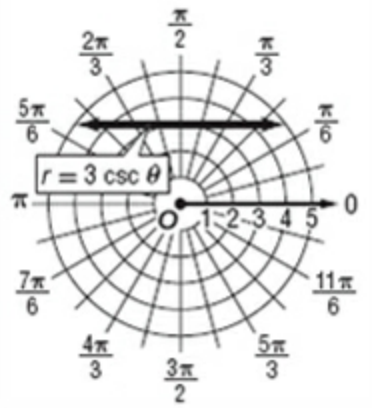
12. $r = 3\csc\theta$

SOLUTION:

Make a table of values to find the r-values corresponding to various values of θ on the interval $[0, 2\pi]$. Round each r-value to the nearest tenth.

θ	$r = 3\csc\theta$
0	–
$\frac{\pi}{6}$	6
$\frac{\pi}{3}$	3.5
$\frac{\pi}{2}$	3
$\frac{2\pi}{3}$	3.5
$\frac{5\pi}{6}$	6
π	–
$\frac{7\pi}{6}$	−6
$\frac{4\pi}{3}$	−3.5
$\frac{3\pi}{2}$	−3
$\frac{5\pi}{3}$	−3.5
$\frac{11\pi}{6}$	−6
2π	–

Graph the ordered pairs (r, θ) and connect them with a line.



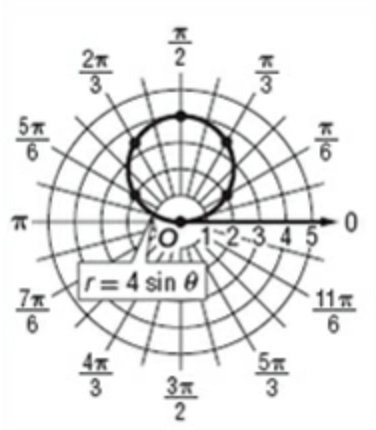
13. $r = 4 \sin \theta$

SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 4 \sin \theta$
$-\frac{\pi}{2}$	–4
$-\frac{\pi}{3}$	–3.5
$-\frac{\pi}{4}$	–2.8
$-\frac{\pi}{6}$	–2
0	0
$\frac{\pi}{6}$	2
$\frac{\pi}{4}$	2.8
$\frac{\pi}{3}$	3.5
$\frac{\pi}{2}$	4

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



14. **STAINED GLASS** A rose window is a circular window seen in gothic architecture. The pattern of the window radiates from the center. The window shown can be approximated by the equation $r = 3 \sin 6\theta$. Use symmetry, zeros, and maximum r-values of the function to graph the function.
Refer to the image on Page 560.

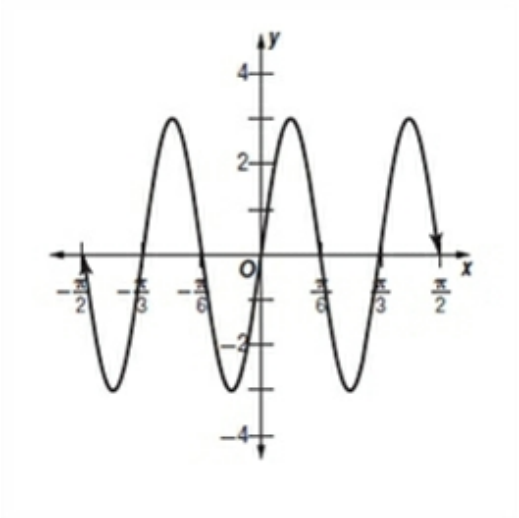
SOLUTION:

Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$.

Sketch the graph of the rectangular function $y = 3 \sin 6x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. From the graph, you can see that $|y| = 3$ when

$x = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \text{and } \frac{5\pi}{12}$ and $y = 0$ when

$x = -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}, \text{and } \frac{\pi}{2}$.



Interpreting these results in terms of the polar equation $r = 3 \sin 6\theta$, we can say that $|r|$ has a maximum value of 3 when

$\theta = -\frac{5\pi}{12}, -\frac{\pi}{4}, -\frac{\pi}{12}, \frac{\pi}{12}, \frac{\pi}{4}, \text{or } \frac{5\pi}{12}$ and $r = 0$ when

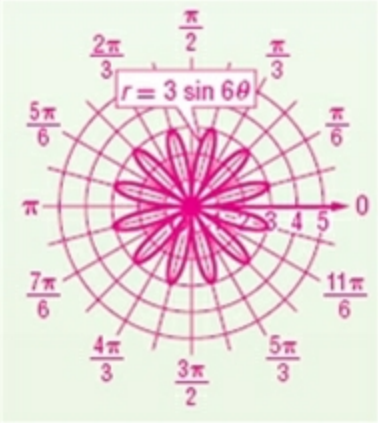
$\theta = -\frac{\pi}{2}, -\frac{\pi}{3}, -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{3}, \text{or } \frac{\pi}{2}$.

Since the function is symmetric with respect to the line $\theta = \frac{\pi}{2}$, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 3 \sin 6\theta$

$-\frac{3\pi}{8}$	−2.1
$-\frac{\pi}{4}$	3
$-\frac{\pi}{8}$	−2.1
0	0
$\frac{\pi}{8}$	2.1
$\frac{\pi}{4}$	−3
$\frac{3\pi}{8}$	2.1

Use these and a few additional points to sketch the graph of the function.



Identify and graph each classic curve.

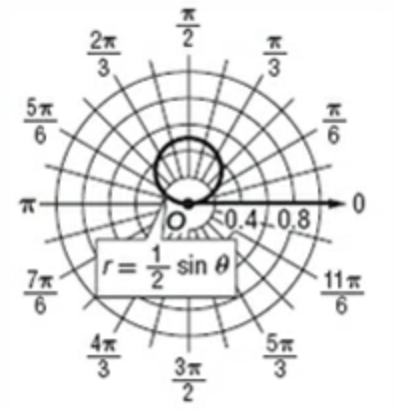
15. $r = \frac{1}{2}\sin\theta$

SOLUTION:

The equation is of the form $r = a \sin \theta$, so its graph is a circle. Because the polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = \frac{1}{2}\sin\theta$
$-\frac{\pi}{2}$	−0.5
$-\frac{\pi}{3}$	−0.4
$-\frac{\pi}{4}$	−0.4
$-\frac{\pi}{6}$	−0.3
0	0
$\frac{\pi}{6}$	0.3
$\frac{\pi}{4}$	0.4
$\frac{\pi}{3}$	0.4
$\frac{\pi}{2}$	0.5

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



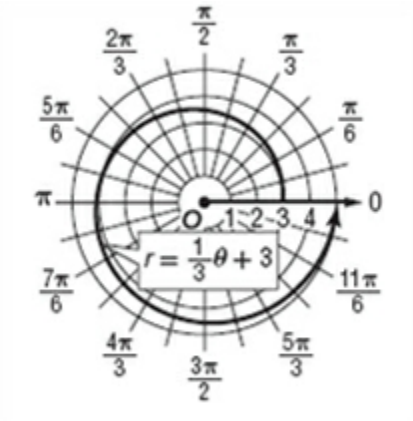
16. $r = \frac{1}{3}\theta + 3,\ \theta \geq 0$

SOLUTION:

The equation is of the form $r = a\theta + b$, so its graph is a spiral of Archimedes.

Use points on the interval [0, 2π] to sketch the graph of the function.

θ	$r = \frac{1}{3}\theta + 3$
0	3
$\frac{\pi}{4}$	3.3
$\frac{\pi}{2}$	3.5
π	4.0
$\frac{3\pi}{2}$	4.6
2π	5.1



17. $r = 1 + 2\cos\theta$

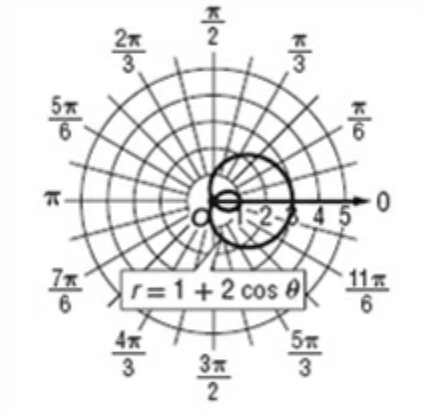
SOLUTION:

The equation is of the form $r = a + b\cos\theta$, so its graph is a limacon. Since $a < b$, the graph with have an inner loop. Because this polar equation is a function of the cosine function, it is symmetric with respect to the polar axis.

Therefore, make a table and calculate the values of r on $[0, \pi]$.

θ	$r = 1 + 2\cos\theta$
0	3
$\frac{\pi}{6}$	2.7
$\frac{\pi}{4}$	2.4
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	0
$\frac{3\pi}{4}$	−0.4
$\frac{5\pi}{6}$	−0.7
π	−1

Use these points and polar axis symmetry to graph the function.



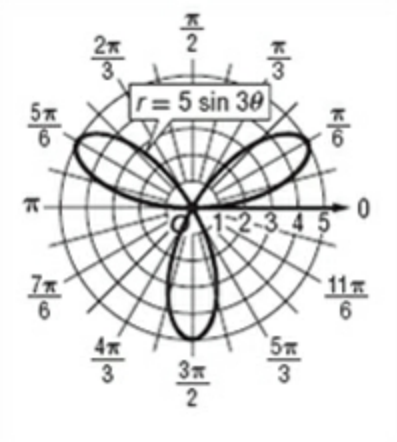
18. $r = 5 \sin 3\theta$

SOLUTION:

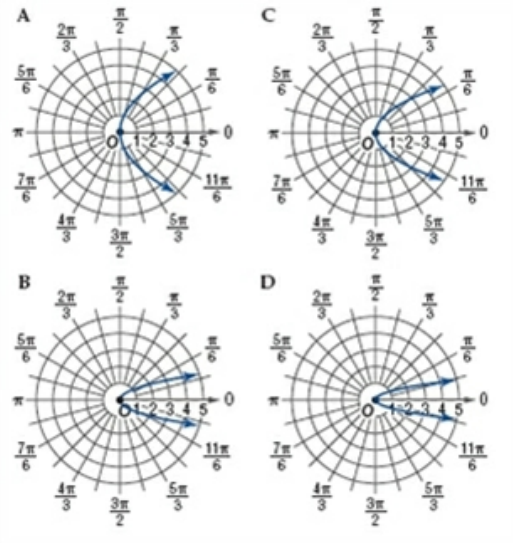
The equation is of the form $r = a \sin n\theta$, so its graph is a rose. Because this polar equation is a function of the sine function, it is symmetric with respect to the line $\theta = \frac{\pi}{2}$. Therefore, make a table and calculate the values of r on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

θ	$r = 5 \sin 3\theta$
$-\frac{\pi}{2}$	5
$-\frac{\pi}{3}$	0
$-\frac{\pi}{4}$	−3.5
$-\frac{\pi}{6}$	−5
0	0
$\frac{\pi}{6}$	5
$\frac{\pi}{4}$	3.5
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	−5

Use these points and symmetry with respect to the line $\theta = \frac{\pi}{2}$ to graph the function.



19. **MULTIPLE CHOICE** Identify the polar graph of $y^2 = \frac{1}{2}x$.



SOLUTION:

Write the rectangular equation $y^2 = \frac{1}{2}x$ in polar form.

$$y^2 = \frac{1}{2}x$$
$$(r\sin\theta)^2 = \frac{1}{2}(r\cos\theta)$$
$$r^2\sin^2\theta = \frac{1}{2}r\cos\theta$$
$$r\sin^2\theta = \frac{1}{2}\cos\theta$$
$$r = \frac{\cos\theta}{2\sin^2\theta}$$
$$r = \frac{1}{2}\cos\theta\csc^2\theta$$

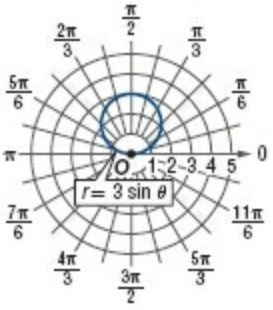
Graph $r = \frac{1}{2}\cos\theta\csc^2\theta$ using a graphing calculator. Let $\theta = \frac{\pi}{6}$ and solve for r.

$$r = \frac{1}{2}\cos\theta\csc^2\theta$$
$$r = \frac{1}{2}\cos\frac{\pi}{6}\csc^2\frac{\pi}{6}$$
$$r \approx 1.73$$

The point $\left(1.73, \frac{\pi}{6}\right)$ corresponds to graph B. The correct answer is B.

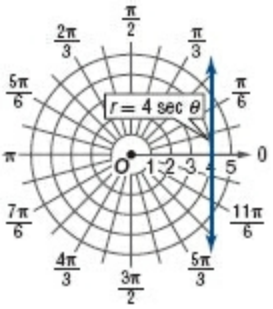
Mid-Chapter Quiz: Lessons 9-1 through 9-3

Write a rectangular equation for each graph.

28. 

SOLUTION:

$$r = 3\sin\theta$$
$$r^2 = 3r\sin\theta$$
$$x^2 + y^2 = 3y$$
$$x^2 + y^2 - 3y = 0$$
$$x^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

29. 

SOLUTION:

$$r = 4\sec\theta$$
$$r = \frac{4}{\cos\theta}$$
$$r\cos\theta = 4$$
$$x = 4$$