Practice Test - Chapter 5

Find the value of each expression using the given information.

1. sin θ and cos θ, csc θ = –4, cos $\theta < 0$

SOLUTION:

Use the reciprocal identity sin $\theta = \frac{1}{\csc\theta}$ to find sin θ.

$$\sin\theta = \frac{1}{\csc\theta} = -\frac{1}{4}$$

Use the Pythagorean Identity that involves sin θ to find cos θ.

$$\sin^2\theta + \cos^2\theta = 1$$
$$\left(-\frac{1}{4}\right)^2 + \cos^2\theta = 1$$
$$\frac{1}{16} + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - \frac{1}{16}$$
$$\cos^2\theta = \frac{15}{16}$$
$$\cos\theta = \pm\sqrt{\frac{15}{16}}$$
$$\cos\theta = \pm\frac{\sqrt{15}}{4}$$

Since cos $\theta < 0$, $\cos\theta = -\frac{\sqrt{15}}{4}$.

2. csc θ and sec θ, tan $\theta = \frac{2}{5}$, csc $\theta < 0$

SOLUTION:

Use the Pythagorean Identity that involves tan θ to find sec θ.

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\left(\frac{2}{5}\right)^2 + 1 = \sec^2\theta$$
$$\frac{4}{25} + 1 = \sec^2\theta$$
$$\frac{29}{25} = \sec^2\theta$$
$$\pm\sqrt{\frac{29}{25}} = \sec\theta$$
$$\pm\frac{\sqrt{29}}{5} = \sec\theta$$

Since csc $\theta < 0$, sin θ is negative. Since tan $\theta = \frac{\sin\theta}{\cos\theta}$ is positive and sin θ is negative, cos θ is negative. Since cos θ is negative, sec θ must be negative. So, $-\frac{\sqrt{29}}{5} = \sec\theta$.

Use the reciprocal identity cos $\theta = \frac{1}{\sec\theta}$ to find cos θ.

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{-\frac{\sqrt{29}}{5}} = -\frac{5}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

Use the Pythagorean Identity that involves cos θ to find sin θ.

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$$\sin^2\theta+\cos^2\theta=1$$
$$\sin^2\theta+\left(-\frac{5\sqrt{29}}{29}\right)^2=1$$
$$\sin^2\theta+\frac{25}{29}=1$$
$$\sin^2\theta=1-\frac{25}{29}$$
$$\sin^2\theta=\frac{4}{29}$$
$$\sin\theta=\pm\sqrt{\frac{4}{29}}$$
$$\sin\theta=\pm\frac{2}{\sqrt{29}}$$
$$\sin\theta=\pm\frac{2\sqrt{29}}{29}$$

Since sin θ is negative, $\sin\theta=-\frac{2\sqrt{29}}{29}$. Use the reciprocal identity csc θ = $\frac{1}{\sin\theta}$ to find csc θ.

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\left(-\frac{2\sqrt{29}}{29}\right)}=-\frac{29}{2\sqrt{29}}=-\frac{\sqrt{29}}{2}$$

Simplify each expression.

3. $\frac{\sin(90°-x)}{\tan(90°-x)}$

SOLUTION:

$$\frac{\sin(90-x)}{\tan(90-x)}=\frac{\cos x}{\cot x}=\frac{\cos x}{\frac{\cos x}{\sin x}}=\cos x\cdot\frac{\sin x}{\cos x}=\sin x$$

4. $\frac{\sec^2 x-1}{\tan^2 x+1}$

SOLUTION:

$$\frac{\sec^2 x-1}{\tan^2 x+1}=\frac{\tan^2 x}{\sec^2 x}=\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}=\frac{\sin^2 x}{\cos^2 x}\cdot\cos^2 x=\sin^2 x$$

5. sin θ (1 + cot^2 θ)

SOLUTION:

$$\sin\theta(1+\cot^2\theta)=\sin\theta\csc^2\theta=\sin\theta\frac{1}{\sin^2\theta}=\frac{1}{\sin\theta}=\csc\theta$$

Verify each identity.

6. $\frac{\csc^2\theta-1}{\csc^2\theta}+\frac{\sec^2\theta-1}{\sec^2\theta}=1$

SOLUTION:

$\frac{\csc^2\theta-1}{\csc^2\theta}+\frac{\sec^2\theta-1}{\sec^2\theta}$	
$=\frac{\cot^2\theta}{\csc^2\theta}+\frac{\tan^2\theta}{\sec^2\theta}$	Pythagorean Identities
$=\frac{\cot^2\theta}{\frac{1}{\sin^2\theta}}+\frac{\tan^2\theta}{\frac{1}{\cos^2\theta}}$	Reciprocal Identities
$=\frac{\cos^2\theta}{\sin^2\theta}\cdot\sin^2\theta+\frac{\sin^2\theta}{\cos^2\theta}\cdot\cos^2\theta$	Use Quotient Identities and multiply by the reciprocals.
$=\cos^2\theta+\sin^2\theta$	Simplify.
$=1$	PythagoreanIdentity

Practice Test - Chapter 5

7. $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{1-\sin\theta}{\cos\theta}=\dfrac{2\cos\theta}{1+\sin\theta}$

SOLUTION:

$$\begin{aligned}
&\frac{\cos\theta}{1+\sin\theta}+\frac{1-\sin\theta}{\cos\theta} & \\
&=\frac{\cos\theta}{1+\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}+\frac{1-\sin\theta}{\cos\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta} & \text{Common denominator}\\
&=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}+\frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} & \text{Simplify.}\\
&=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}+\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} & \text{Pythagorean Identity}\\
&=\frac{2\cos^2\theta}{\cos\theta(1+\sin\theta)} & \text{Add.}\\
&=\frac{2\cos\theta}{1+\sin\theta} & \text{Divide numerator and denominator by } \cos\theta.
\end{aligned}$$

8. $\dfrac{1}{1+\cos\theta}+\dfrac{1}{1-\cos\theta}=2\csc^2\theta$

SOLUTION:

$$\begin{aligned}
&\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta} & \\
&=\frac{1}{1+\cos\theta}\cdot\frac{1-\cos\theta}{1-\cos\theta}+\frac{1}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta} & \text{Common denominator}\\
&=\frac{1-\cos\theta}{1-\cos^2\theta}+\frac{1+\cos\theta}{1-\cos^2\theta} & \text{Simplify.}\\
&=\frac{2}{\sin^2\theta} & \text{Use the Pythagorean Identity and add.}\\
&=2\csc^2\theta & \text{Reciprocal Identity}
\end{aligned}$$

9. $-\sec^2\theta\sin^2\theta=\dfrac{\cos^2\theta-1}{\cos^2\theta}$

SOLUTION:

$$\begin{aligned}
&-\sec^2\theta\sin^2\theta & \\
&=\sec^2\theta(-\sin^2\theta) & \text{Commutative Property}\\
&=\frac{1}{\cos^2\theta}(-\sin^2\theta) & \text{Reciprocal Identity}\\
&=\frac{-\sin^2\theta}{\cos^2\theta} & \text{Multiply.}\\
&=\frac{-(1-\cos^2\theta)}{\cos^2\theta} & \text{Pythagorean Identity}\\
&=\frac{\cos^2\theta-1}{\cos^2\theta} & \text{Simplify.}
\end{aligned}$$

10. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

SOLUTION:

$$\begin{aligned}
&\sin^4 x-\cos^4 x & \\
&=(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x) & \text{Factor.}\\
&=(\sin^2 x-\cos^2 x)\cdot 1 & \text{Pythagorean Identity}\\
&=[\sin^2 x-(1-\sin^2 x)] & \text{Pythagorean Identity}\\
&=\sin^2 x-1+\sin^2 x & \text{Simplify.}\\
&=2\sin^2 x-1 & \text{Add like terms.}
\end{aligned}$$

11. **MULTIPLE CHOICE** Which expression is *not* true?

A $\tan(-\theta) = -\tan\theta$

B $\tan(-\theta) = \dfrac{1}{\cot(-\theta)}$

C $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)}$

D $\tan(-\theta) + 1 = \sec(-\theta)$

SOLUTION:

Choice A is true due to the Odd-Even Identity. Choice B is true due to the Reciprocal Identity. Choice C is true due to the Quotient Identity. Verify Choice D using the quotient and reciprocal identities.

$$\begin{aligned}
\tan(-\theta)+1 &\stackrel{?}{=}\sec(-\theta)\\
\frac{\sin(-\theta)}{\cos(-\theta)}+1 &\stackrel{?}{=}\frac{1}{\cos(-\theta)}\\
\frac{\sin(-\theta)}{\cos(-\theta)}+\frac{\cos(-\theta)}{\cos(-\theta)} &\stackrel{?}{=}\frac{1}{\cos(-\theta)}\\
\frac{\sin(-\theta)+\cos(-\theta)}{\cos(-\theta)} &\neq\frac{1}{\cos(-\theta)}
\end{aligned}$$

Choice D is not true.
The correct answer is D.

Find all solutions of each equation on the interval [0, 2π].

12. $\sqrt{2}\sin\theta + 1 = 0$

SOLUTION:

$$\begin{aligned}
\sqrt{2}\sin\theta+1&=0\\
\sqrt{2}\sin\theta&=-1\\
\sin\theta&=-\frac{1}{\sqrt{2}}\\
\sin\theta&=-\frac{\sqrt{2}}{2}
\end{aligned}$$

On the interval [0, 2π), $\sin\theta=-\dfrac{\sqrt{2}}{2}$ when $\theta = \dfrac{5\pi}{4}$ and $\theta=\dfrac{7\pi}{4}$.

13. $\sec^2\theta = \frac{4}{3}$

SOLUTION:

$$\sec^2\theta = \frac{4}{3}$$
$$\sec\theta = \pm\sqrt{\frac{4}{3}}$$
$$\sec\theta = \pm\frac{2}{\sqrt{3}}$$
$$\sec\theta = \pm\frac{2\sqrt{3}}{3}$$

On the interval [0, 2π), $\sec\theta = \frac{2\sqrt{3}}{3}$ when $\theta = \frac{\pi}{6}$ and $\theta = \frac{11\pi}{6}$ and $\sec\theta = -\frac{2\sqrt{3}}{3}$ when $\theta = \frac{5\pi}{6}$ and $\theta = \frac{7\pi}{6}$.

Solve each equation for all values of θ.

14. $\tan^2\theta - \tan\theta = 0$

SOLUTION:

$$\tan^2\theta - \tan\theta = 0$$
$$\tan\theta(\tan\theta - 1) = 0$$

$$\tan\theta = 0$$

$$\tan\theta - 1 = 0$$
$$\tan\theta = 1$$

The period of tangent is π, so you only need to find solutions on the interval $[0, \pi)$. The solution to tan θ = 0 on this interval is 0 and the solution to tan θ = 1 on this interval is $\frac{\pi}{4}$.

Solutions on the interval (–∞ , ∞), are found by adding integer multiples of π. Therefore, the general form of the solutions is $n\pi$, $\frac{\pi}{4} + n\pi$, where n is an integer.

15. $\frac{1 - \sin\theta}{\cos\theta} = \cos\theta$

SOLUTION:

$$\frac{1-\sin\theta}{\cos\theta} = \cos\theta$$
$$1 - \sin\theta = \cos^2\theta$$
$$1 - \sin\theta = 1 - \sin^2\theta$$
$$\sin^2\theta - \sin\theta = 0$$
$$\sin\theta(\sin\theta - 1) = 0$$

$$\sin\theta = 0$$

$$\sin\theta - 1 = 0$$
$$\sin\theta = 1$$

The period of sine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions to sin θ = 0 on this interval are 0 and π and the solution to sin θ = 1 on this interval is $\frac{\pi}{2}$. However, since cos $\frac{\pi}{2}$ = 0, the equation is undefined for $\theta = \frac{\pi}{2}$.

Solutions on the interval (–∞ , ∞), are found by adding integer multiples of 2π. The solutions $\theta = 0 + 2n\pi$ and $\theta = \pi + 2n\pi$ can be combined to $\theta = n\pi$. Therefore, the general form of the solutions is $n\pi$, where n is an integer.

16. $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2$

SOLUTION:

$$\frac{1}{\sec\theta-1} - \frac{1}{\sec\theta+1} = 2$$
$$\frac{\sec\theta+1}{\sec^2\theta-1} - \frac{\sec\theta-1}{\sec^2\theta-1} = 2$$
$$\frac{2}{\sec^2\theta-1} = 2$$
$$\frac{2}{\tan^2\theta} = 2$$
$$2 = 2\tan^2\theta$$
$$1 = \tan^2\theta$$
$$\pm 1 = \tan\theta$$

The period of tangent is π, so you only need to find solutions on the interval $[0, \pi)$. The solution to tan θ = 1 on this interval is $\frac{\pi}{4}$ and the solution to tan θ = −1 on this interval is $\frac{3\pi}{4}$.

Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of π. The solutions $\frac{\pi}{4} + n\pi$ and $\frac{3\pi}{4} + n\pi$ can be combined to $\frac{\pi}{4} + \frac{\pi}{2}n$. Therefore, the general form of the solutions is $\frac{\pi}{4} + \frac{\pi}{2}n$, where n is an integer.

17. sec θ – 2tan θ = 0

SOLUTION:

$$\sec\theta - 2\tan\theta = 0$$
$$\frac{1}{\cos\theta} - \frac{2\sin\theta}{\cos\theta} = 0$$
$$\frac{1-2\sin\theta}{\cos\theta} = 0$$
$$1 - 2\sin\theta = 0$$
$$-2\sin\theta = -1$$
$$\sin\theta = \frac{1}{2}$$

The period of sine is 2π, so you only need to find solutions on the interval $[0, 2\pi)$. The solutions to sin $\theta = \frac{1}{2}$ on this interval are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. The equation is undefined when cos θ = 0, so these values of θ are not part of the solution.

Solutions on the interval $(-\infty, \infty)$, are found by adding integer multiples of 2π. Therefore, the general form of the solutions is $\frac{\pi}{6} + 2n\pi$, $\frac{5\pi}{6} + 2n\pi$, where n is an integer.

18. **CURRENT** The current produced by an alternator is given by $I = 40 \sin 135\pi t$, where I is the current in amperes and t is the time in seconds. At what time t does the current first reach 20 amperes? Round to the nearest ten-thousandths.

SOLUTION:

Substitute $I = 20$ into $I = 40 \sin 135\pi t$ and solve for t.

$$I = 40\sin 135\pi t$$
$$20 = 40\sin 135\pi t$$
$$\frac{1}{2} = \sin 135\pi t$$
$$\sin^{-1}\frac{1}{2} = 135\pi t$$
$$\frac{\pi}{6} = 135\pi t$$
$$\frac{1}{810} = t$$
$$0.0012 \approx t$$

The time at which the current first reaches 20 amperes is about 0.0012 second.

Practice Test - Chapter 5

Find the exact value of each trigonometric expression.

19. tan 165°

SOLUTION:

Write 165° as the sum or difference of angle measures with tangents that you know.

$$\begin{aligned}\tan 165^\circ &= \tan(120^\circ + 45^\circ)\\ &= \frac{\tan 120^\circ + \tan 45^\circ}{1 - \tan 120^\circ \tan 45^\circ}\\ &= \frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)}\\ &= \frac{1-\sqrt{3}}{1+\sqrt{3}}\\ &= \frac{1-\sqrt{3}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}\\ &= \frac{4-2\sqrt{3}}{-2}\\ &= -2+\sqrt{3}\end{aligned}$$

20. $\cos -\frac{\pi}{12}$

SOLUTION:

Write $-\frac{\pi}{12}$ as the sum or difference of angle measures with cosines that you know.

$$\begin{aligned}\cos -\frac{\pi}{12} &= \cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right)\\ &= \cos\frac{\pi}{6}\cos\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{\pi}{4}\\ &= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

21. sin 75°

SOLUTION:

Write 75° as the sum or difference of angle measures with sines that you know.

$$\begin{aligned}\sin 75^\circ &= \sin(45^\circ + 30^\circ)\\ &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ\\ &= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

22. cos 465° – cos 15°

SOLUTION:

$$\begin{aligned}\cos 465^\circ - \cos 15^\circ &= -2\sin\left(\frac{465^\circ+15^\circ}{2}\right)\sin\left(\frac{465^\circ-15^\circ}{2}\right)\\ &= -2\sin 240^\circ \sin 225^\circ\\ &= -2\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)\\ &= -\frac{\sqrt{6}}{2}\end{aligned}$$

23. 6 sin 675° – 6 sin 45°

SOLUTION:

$$\begin{aligned}6\sin 675^\circ - 6\sin 45^\circ &= 6(\sin 675^\circ - \sin 45^\circ)\\ &= 6\left[2\cos\left(\frac{675^\circ+45^\circ}{2}\right)\sin\left(\frac{675^\circ-45^\circ}{2}\right)\right]\\ &= 6(2\cos 360^\circ \sin 315^\circ)\\ &= 6\left[2(1)\left(-\frac{\sqrt{2}}{2}\right)\right]\\ &= -6\sqrt{2}\end{aligned}$$

Practice Test - Chapter 5

24. **MULTIPLE CHOICE** Which identity is true?

F $\cos(\theta + \pi) = -\sin \pi$

G $\cos(\pi - \theta) = \cos \theta$

H $\sin\left(\theta - \frac{3\pi}{2}\right) = \cos \theta$

J $\sin(\pi + \theta) = \sin \theta$

SOLUTION:

Use the sum or difference identities to verify each identity.

Choice F:

$$\begin{aligned}\cos(\theta+\pi) &= \cos\theta\cos\pi - \sin\theta\sin\pi \\ &= \cos\theta(-1) - \sin\theta(0) \\ &= -\cos\theta\end{aligned}$$

Choice F is false.

Choice G:

$$\begin{aligned}\cos(\pi-\theta) &= \cos\pi\cos\theta + \sin\pi\sin\theta \\ &= (-1)\cos\theta + (0)\sin\theta \\ &= -\cos\theta\end{aligned}$$

Choice G is false.

Choice H:

$$\begin{aligned}\sin\left(\theta-\frac{3\pi}{2}\right) &= \sin\theta\cos\frac{3\pi}{2} - \cos\theta\sin\frac{3\pi}{2} \\ &= \sin\theta(0) - \cos\theta(-1) \\ &= \cos\theta\end{aligned}$$

Choice H is true.

Choice J:

$$\begin{aligned}\sin(\pi+\theta) &= \sin\pi\cos\theta + \cos\pi\sin\theta \\ &= (0)\cos\theta + (-1)\sin\theta \\ &= -\sin\theta\end{aligned}$$

Choice J is false.

The correct answer is H.

Simplify each expression.

25. $\cos \frac{\pi}{8} \cos \frac{3\pi}{8} - \sin \frac{\pi}{8} \sin \frac{3\pi}{8}$

SOLUTION:

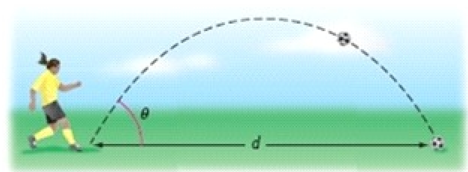
$$\begin{aligned}\cos\frac{\pi}{8}\cos\frac{3\pi}{8} - \sin\frac{\pi}{8}\sin\frac{3\pi}{8} &= \cos\left(\frac{\pi}{8}+\frac{3\pi}{8}\right) \\ &= \cos\frac{\pi}{2} \\ &= 0\end{aligned}$$

26. $\frac{\tan 135° - \tan 15°}{1 + \tan 135° \tan 15°}$

SOLUTION:

$$\begin{aligned}\frac{\tan 135° - \tan 15°}{1+\tan 135° \tan 15°} &= \tan(135° - 15°) \\ &= \tan 120° \\ &= -\sqrt{3}\end{aligned}$$

27. **PHYSICS** A soccer ball is kicked from ground level with an initial speed of v at an angle of elevation θ.



a. The horizontal distance d the ball will travel can be determined using $d = \frac{v^2 \sin 2\theta}{g}$, where g is the acceleration due to gravity.

Verify that this expression is the same as $\frac{2}{g}v^2(\tan \theta - \tan\theta \sin^2 \theta)$.

b. The maximum height h the object will reach can be determined using $h = \frac{v^2 \sin^2 \theta}{2g}$. Find the ratio of the maximum height attained to the horizontal distance traveled.

SOLUTION:

a.

$\frac{2}{g}v^2(\tan\theta - \tan\theta\sin^2\theta)$

$= \frac{2}{g}v^2\tan\theta(1-\sin^2\theta)$ Factor $\tan\theta$ from each term.

$= \frac{2}{g}v^2\tan\theta\cos^2\theta$ Pythagorean Identity

$= \frac{2}{g}v^2\frac{\sin\theta}{\cos\theta}\cos^2\theta$ Quotient Identity

$= \frac{2}{g}v^2\sin\theta\cos\theta$ Simplify.

$= \frac{v^2\sin 2\theta}{g}$ Double-Angle Identity

b. To find the ratio of the maximum height attained to the horizontal distance traveled, find $\frac{h}{d}$.

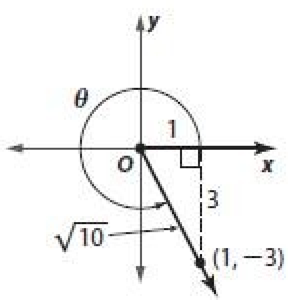
$$\frac{h}{d}=\frac{\frac{v^2\sin^2\theta}{2g}}{\frac{v^2\sin 2\theta}{g}}$$
$$=\frac{v^2\sin^2\theta}{2g}\cdot\frac{g}{v^2\sin 2\theta}$$
$$=\frac{\sin^2\theta}{2\sin 2\theta}$$
$$=\frac{\sin^2\theta}{2(2\sin\theta\cos\theta)}$$
$$=\frac{\sin\theta}{4\cos\theta}$$
$$=\frac{1}{4}\tan\theta$$

Find the values of sin 2θ, cos 2θ, and tan 2θ for the given value and interval.

28. tan θ = –3, $\left(\frac{3\pi}{2}, 2\pi\right)$

SOLUTION:

If tan θ = –3, then $\tan\theta=-\frac{3}{1}$. Since $\tan\theta=-\frac{3}{1}$ on the interval $\left(\frac{3\pi}{2}, 2\pi\right)$, one point on the terminal side of θ has x-coordinate 1 and y-coordinate –3 as shown. The distance from the point to the origin is $\sqrt{(-3)^2+1^2}$ or $\sqrt{10}$.



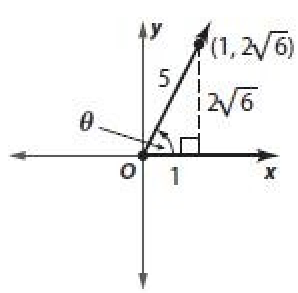
Using this point, we find that $\sin\theta=\frac{y}{r}$ or $-\frac{3\sqrt{10}}{10}$ and $\cos\theta=\frac{x}{r}$ or $\frac{\sqrt{10}}{10}$. Now use the double-angle identities for sine, cosine, and tangent to find sin 2θ cos 2θ, and tan 2θ.

$$\sin 2\theta=2\sin\theta\cos\theta$$
$$=2\left(-\frac{3\sqrt{10}}{10}\right)\left(\frac{\sqrt{10}}{10}\right)$$
$$=-\frac{3}{5}$$
$$\cos 2\theta=2\cos^2\theta-1$$
$$=2\left(\frac{\sqrt{10}}{10}\right)^2-1$$
$$=2\left(\frac{1}{10}\right)-1$$
$$=\frac{2}{10}-\frac{10}{10}$$
$$=-\frac{4}{5}$$
$$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$$
$$=\frac{2(-3)}{1-(-3)^2}$$
$$=\frac{-6}{1-9}$$
$$=\frac{3}{4}$$

29. cos θ = $\frac{1}{5}$, (0°, 90°)

SOLUTION:

Since $\cos\theta=\frac{1}{5}$ on the interval (0°, 90°), one point on the terminal side of θ has x-coordinate 1 and a distance of 5 units from the origin as shown. The y-coordinate of this point is therefore $\sqrt{5^2-1^2}$ or $2\sqrt{6}$.



Using this point, we find that $\sin\theta = \frac{y}{r}$ or $\frac{2\sqrt{6}}{5}$ and $\tan\theta = \frac{y}{x}$ or $2\sqrt{6}$. Now use the double-angle identities for sine, cosine, and tangent to find sin 2θ, cos 2θ, and tan 2θ.

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= 2\left(\frac{2\sqrt{6}}{5}\right)\left(\frac{1}{5}\right)\\ &= \frac{4\sqrt{6}}{25}\end{aligned}$$

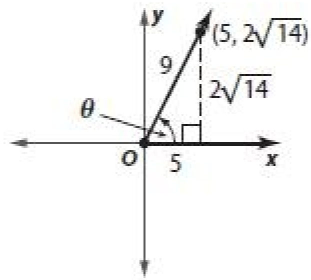
$$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1\\ &= 2\left(\frac{1}{5}\right)^2 - 1\\ &= 2\left(\frac{1}{25}\right) - 1\\ &= \frac{2}{25} - \frac{25}{25}\\ &= -\frac{23}{25}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\\ &= \frac{2(2\sqrt{6})}{1-(2\sqrt{6})^2}\\ &= \frac{4\sqrt{6}}{1-24}\\ &= -\frac{4\sqrt{6}}{23}\end{aligned}$$

30. $\cos\theta = \frac{5}{9}, \left(0, \frac{\pi}{2}\right)$

SOLUTION:

Since $\cos\theta = \frac{5}{9}$ on the interval $\left(0, \frac{\pi}{2}\right)$, one point on the terminal side of θ has *x*-coordinate 5 and a distance of 9 units from the origin as shown. The *y*-coordinate of this point is therefore $\sqrt{9^2 - 5^2}$ or $2\sqrt{14}$.



Using this point, we find that $\sin\theta = \frac{y}{r}$ or $\frac{2\sqrt{14}}{9}$ and $\tan\theta = \frac{y}{x}$ or $\frac{2\sqrt{14}}{5}$. Now use the double-angle identities for sine, cosine, and tangent to find sin 2θ, cos 2θ, and tan 2θ.

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ &= 2\left(\frac{2\sqrt{14}}{9}\right)\left(\frac{5}{9}\right)\\ &= \frac{20\sqrt{14}}{81}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= 2\cos^2\theta - 1\\ &= 2\left(\frac{5}{9}\right)^2 - 1\\ &= 2\left(\frac{25}{81}\right) - 1\\ &= \frac{50}{81} - \frac{81}{81}\\ &= -\frac{31}{81}\end{aligned}$$

$$\begin{aligned}\tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\\ &= \frac{2\left(\frac{2\sqrt{14}}{5}\right)}{1-\left(\frac{2\sqrt{14}}{5}\right)^2}\\ &= \frac{\frac{4\sqrt{14}}{5}}{1-\frac{56}{25}}\\ &= \frac{\frac{4\sqrt{14}}{5}}{-\frac{31}{25}}\\ &= -\frac{20\sqrt{14}}{31}\end{aligned}$$